MATERIAL VOIDS IN ELASTIC SOLIDS: A SECOND ORDER MINIMALITY CRITERION

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Abstract

In this talk we consider a two-dimensional variational model that has been recently used to describe surface instability in morphological evolution of cavities in stressed solids. Our aim is to describe how to perform a second order variational analysis in order to obtain a new minimality conditions for equilibria and studying their stability.

Consider a cavity in an elastic solid, that will be identified with a smooth compact set $F \subset \mathbb{R}^2$, starshaped with respect to the origin. The solid region is assumed to obey to the classical law of linear elasticity, so that the bulk energy can be written in the form

$$\int_{B\setminus F} Q(E(u))\, dz,$$

where $E(u)$ is the symmetric gradient of the elastic displacement u and Q is a bilinear form depending on the material. The surface energy is simply assumed to be the length of the boundary of F. Then the energy is expressed by the functional

$$\mathcal{F}(F,u) := \int_{B\setminus F} Q(E(u))\, dz + \mathcal{H}^1(\partial F)\,.$$

The equilibria are identified with minimizers of $\mathcal{F}(F,u)$ under the volume constraint $|F| = d$. We will present a quantitative minimality criterion that relies on the study of the second variation of the functional $\mathcal{F}$.